

Description

CURVED GRATING SPECTROMETER WITH VERY HIGH WAVELENGTH RESOLUTION

CROSS REFERENCE TO RELATED APPLICATIONS

[0001] This application is related to U.S. Patent Application Serial No. 09/916701, "Method for shifting the bandgap energy of quantum well layer" filed July 26, 2001, in the names of Boon-Siew Ooi and Seng-Tiong Ho, the disclosure of which is incorporated herein by reference. This application is related to U.S. Patent Application Serial No. 60/242219, "Method for shifting the bandgap energy of quantum well layer" filed October 20, 2000, in the names of Boon-Siew Ooi and Seng-Tiong Ho, the disclosure of which is incorporated herein by reference. This application is related to U.S. Patent Application Serial No. 60/430507, "Method for quantum-well intermixing using pre-annealing enhanced defect diffusion" filed December 3, 2002, in the names of Boon-Siew Ooi and Ruiyu (Jane) Wang, the disclosure of which is incorporated herein by reference.

BACKGROUND OF INVENTION

[0002] The present invention relates to semiconductor photonic and opto-electronic devices. In particular, the present invention relates to an integrated optically amplified photo-detector and method of making the same.

[0003] Optical gratings are well known in the art and are used to disperse optical spectra spatially. Such gratings are commonly used in optical spectrometers to analyze the spectra composition of an optical beam. There is always a trade off between the length of an optical spectrometer and its resolution. Thus, if a higher wavelength resolution is required, the length required is also longer. Consider an example of a typical 1-meter long grating spectrometer in the market, which has a wavelength resolution of about $\Delta\lambda=0.1\text{nm}$ at $\lambda=1000\text{nm}$ or $\Delta\lambda/\lambda=10^{-4}$. The dimensionless quantity for the length of the spectrometer L is L/λ and $L/\lambda = 10^6$ in this example. The dimensionless product of the relative resolution $\Delta\lambda/\lambda$ and the relative physical size L/λ of the spectrometer is dependant on the design of the spectrometer and in this example, spectrometer gives $(\Delta\lambda/\lambda) \times (L/\lambda) = 100=RS$. This factor (RS) is generally referred to as the "resolution vs size" factor. RS basically measures the compactness of a spectrometer for a

given resolution power. The smaller the RS value, the more compact is the spectrometer. Only a few conventional spectrometers have RS factor less than about 10. This is primarily because of the various limitations in the current art (as will be described below).

[0004] It is known in the art that a relatively compact spectrometer can be achieved using a curved grating. The schematics of such a grating spectrometer is shown in FIG. 1A, illustrating an optical beam 1 entering an entrance slit 2 with slit size w_1 . The beam, after slit 2, undergoes wave diffraction towards a curved grating 3, which diffracts the beam spatially in a direction that is dependant on the optical wavelength of the beam. The curvature of the grating helps to refocus the diffracted beam to an exit slit 4 with slit size w_2 . Light through slit 4 is then detected by a photo detector 5. As is well known to those skilled in the art, the commonly used design for the curved grating 3 is the Rowland design. In the Rowland design, the grating has a circularly curved shape of radius R 6 and the slits SL1 and SL2 lie in a circle of radius $R/2$ 7 as shown in Figure 1A. The grating is ruled using a diamond tip with constant horizontal displacement d , which ruled the curved surface with constant Chord lines C1, C2, C3 and so forth,

as shown in FIG. 1B. The segment lengths, S1, S2, S3 and so forth, along the curved surface are not a constant and vary along the curved surface.

[0005] Let the diffraction full angle from the entrance slit 1 be θ_{div} . As is well known to those skilled in the art, $\theta_{div} = 2\lambda/d$ (in Radian). Let length L be the distance between the grating center and entrance slit 1, which is also approximately the distance between the grating center and the exit slit 4. As is well known to those skilled in the art, the resolution of the spectrometer increases with decreasing slit size w_2 . The imaging through the curved grating requires w_1 and w_2 to be about equal. A smaller slit size w_1 , however, leads to a larger diffraction angle θ_{div} . It can be shown that the Rowland design works reasonably well up to $\theta_{div} < 4^\circ$. When $\theta_{div} > 4^\circ$, the Rowland design could not give a sharp enough focus at the exit slit 4 (for $\Delta\lambda < 0.1\text{nm}$), thereby limiting the size of w_2 and hence the resolution of the spectrometer. A diffraction angle of $\theta_{div} = 4^\circ$ corresponds to a slit size of about 25 microns (for $\lambda = 1000\text{nm}$). In the current art, it is typically difficult to make slit size smaller than 25 microns, and Rowland design is adequate for most present spectrometers with slit sizes larger than 25 microns.

[0006] *Aberration limitation:* In the case of the Rowland design, when $\theta_{\text{div}} > 4\text{DEG}$, serious aberration in the refocusing beam will occur to limit wavelength resolution. This is shown in Fig. 1C illustrating the ray tracing for the typical Rowland–Echelle design at 4, 8, and 16DEG diffraction. The ray tracing will allow us to see potential focusing distortion or aberration. at the exit slit. In the figure, we show the focusing behavior for two sets of rays with wavelengths separated by 0.4nm. From the figure, we see that their focused spots clearly separate when $\theta_{\text{div}} < 4\text{DEG}$. However, when $\theta_{\text{div}} = 8\text{DEG}$, the focused spots began to smear out. There is substantial distortion for the focusing rays when $\theta_{\text{div}} > 4\text{DEG}$. Further simulations based on numerical solutions to Maxwell's wave equations using finite-difference time-domain (FDTD) method also show similar onset of focused spot size distortion at $\theta_{\text{div}} > 4\text{DEG}$. In short, the current designs are close to their resolution-size (RS) limits and cannot be made substantially more compact without losing wavelength resolution.

[0007] As discussed above, a curved-grating spectrometer is well specified by geometric configurations of its components as shown in FIG. 2. First, the location of the entrance slit; this is usually given by an angle θ_1 with respect to the

normal of the grating center and the distance S_1 from the grating center. The center of the grating refers to the part of the grating hit by the center, i.e. high intensity point, of the entrance beam. Second, the locations of the first two grooves at the grating center; these are specified by its location vectors X_1 and $X_{(-1)}$ with respect to the grating center $X_0 = 0$ and its groove spacing (or pitch) $d_1 = |X_1 - X_0|$ and $d_{-1} = |X_{(-1)} - X_0|$. X_1 and $X_{(-1)}$ are located symmetrically opposite to each other with respect to the grating center and therefore $d = d_1 = d_{(-1)}$. A circle can be defined by these three points X_0, X_1 , and X_2 and its radius is referred to as the radius of curvature at the grating center. Third, the location of the exit slit i.e. the location of the detector; this is specified by an angle θ_2 with respect to the normal of the grating center and the distance S_2 from the grating center. For a given operating wavelength center λ_c , the initial groove spacing d is usually chosen to satisfy the diffraction grating formula for a given entrance slit and detector location. The curved grating is further specified by the location of other grooves (specified by its location vector X_i , with respect to the grating center $X_0 = 0$ and the groove spacing d_i from the previous groove given by $d_i = |X_i - X_{i-1}|$. Let the total number of groove be

N in each side of the grating center, the locus of all the groove $X_{(-N)}, \dots, X_{(-1)}, X_0, X_1, \dots, X_N$ form a curved shape, which can lie in a circle or in any other curvilinear line.

Curved shape of the grating acts as an imaging element of the spectrometer.

[0008] The shape of each groove centered at X_i is not critical to the resolution power of the grating and hence is not necessary to be a part of the main specification. However, the groove shape is related to the diffraction efficiency. For example, in order to increase the diffraction efficiency at a particular diffraction angle θ_2 , it is typically made a planar surface for each groove, oriented in such a way that it acts like a tiny mirror reflecting the input ray towards the angle θ_2 , a process typically referred to as blazing to angle θ_2 (for a given wavelength λ). A section of each groove which reflects light is physically a two-dimensional surface of a particular shape, not a one-dimensional curve. However, the geometric shape of a groove is usually referred to as a curve of a particular shape. This is because there is no variation in the grating shape in the direction perpendicular to the plane where grating lies. Especially, spectrometers within a planar waveguide are strictly two-dimensional in their nature and the shape of grating or

grooves will be referred with a curve, not with surface.

[0009] Conventional Rowland design spectrometers are specifically configured by the design rule described below in conjunction with FIG. 3.

[0010] Referring to FIG. 3, the entrance slit is located on a circle of $R/2$, where R is the radius of curvature at the grating center. This circle of radius $R/2$ is called as Rowland circle and it is tangent to the grating center. In the Rowland design, the distance S_1 of the entrance slit to the grating center is related to the angle of incidence θ_1 by $S_1 = R \times \cos \theta_1$.

[0011] The detector is also located on the same Rowland circle as the entrance slit. In the Rowland design, the distance S_2 of the detector to the grating center is related to the angle of diffraction θ_2 by $S_2 = R \times \cos \theta_2$.

[0012] The relation between θ_1 , θ_2 , and initial groove spacing d is given by the grating formula,

[0013]
$$d(\sin \theta_2 - \sin \theta_1) = m \lambda_c / n \quad (1)$$

[0014] where m is the diffraction order, n is the refractive index of the medium, λ_c is the center of the operation wavelength. This grating formula is a so-called far-field approximation, which is valid only when S_1 and S_2 are much larger than d .

- [0015] Initial groove positions are $X_0 = (0, 0)$, $X_1 = (d, R - (R^2 - d^2)^{1/2})$ and $X_{-1} = (-d, R - (R^2 - d^2)^{1/2})$. These three initial grooves with position vectors X_0, X_1 , and X_{-1} are located on a circle of radius R and have the initial groove spacing of d along a chord parallel to the grating tangent.
- [0016] All other grooves, specified by its position vector X_i 's, are located on the same circle of radius R defined by the initial three groove positions X_0, X_1 , and X_{-1} . X_i 's are also equally spaced along a chord that is parallel to the tangent of the grating center. In other words, the projection of the displacement vector $X_i - X_{i-1}$ on this chord always has the same length. Specifically, the position vectors of these grooves can be written as $X_i = (di, R - (R^2 - (di)^2)^{1/2})$, and $X_{-i} = (-di, R - (R^2 - (di)^2)^{1/2})$.
- [0017] For example, if the radius of curvature at the grating center is $R = 100\mu\text{m}$, the Rowland circle, where the entrance slit and the detector are located, has the radius of $50\mu\text{m}$. Here, we assume that tangent line at the grating center is parallel to the x -axis. Since the Rowland circle is tangent to the grating center, it circles by passing both the grating center $X_0 = (0, 0)$ and a point $(0, 50)$. If the angle of the entrance slit is $\theta_1 = 45^\circ$, the distance of the entrance slit to the grating center is $S_1 = R \times \cos\theta_1 = 70.71\mu\text{m}$. In terms

of (x, y)-coordinate, the entrance slit is located at (-50, 50). It is well-known that grating is more efficient if the propagation direction of the diffracted light from the grating is parallel and opposite to the propagation direction of the input beam. Such a scheme is known as Littrow configuration and is widely used for a high-efficiency spectrometer. A Littrow configuration in the Rowland design will be equivalent to having the angle of detector being almost equal to the angle of the entrance slit, i.e., $\theta_1 \approx \theta_2$. In order to have Littrow configuration, the groove spacing d at the grating center has to be properly chosen so that it satisfies grating formula Eq. 1. For example, when the center wavelength is 1550nm and the angle of entrance slit is $\theta_1 = 45^\circ$, the diffraction order of $m = 12$ of a grating with the groove spacing of $d = 4.2\mu\text{m}$ at its center propagate toward a detector located at $\theta_2 = 37.37^\circ$, which is close to the Littrow configuration. The detector location can be fine tuned by changing initial groove spacing d . Lower the groove spacing d , larger the detector angle θ_2 . For the groove spacing $d = 4.2\mu\text{m}$ and radius of curvature of $R = 100\mu\text{m}$, the initial three positions of grooves are $X_0 = (0, 0)$, $X_1 = (4.2, 0.088)$, and $X_{-1} = (-4.2, 0.088)$.

[0018] In the Rowland design, other grooves are located such that their spacing is the same along a chord parallel to the grating tangent at the center. Therefore, the position vectors of other grooves are $X_i = (di, R - (R^2 - (di)^2)^{1/2}) = (4.2i, 100 - (100^2 - (4.2i)^2)^{1/2})$, and $X_{-i} = (-di, R - (R^2 - (di)^2)^{1/2}) = (-4.2i, 100 - (100^2 - (4.2i)^2)^{1/2})$. The position vectors of the grooves are listed in the following table for the case of Rowland design with $R=100 \mu\text{m}$, $d=4.2\mu\text{m}$, $m = 12$, $\theta_1 = 45^\circ$, and $\theta_2 = 37.37^\circ$ for an operation wavelength of $\lambda_c = 1550\text{nm}$.

X_{-13}	$(-54.6, 16.221)$
X_{-12}	$(-50.4, 13.630)$
X_{-11}	$(-46.2, 11.312)$
X_{-10}	$(-42, 9.248)$
X_{-9}	$(-37.8, 7.419)$
X_{-8}	$(-33.6, 5.814)$
X_{-7}	$(-29.4, 4.419)$
X_{-6}	$(-25.2, 3.227)$
X_{-5}	$(-21, 2.230)$
X_{-4}	$(-16.8, 1.421)$
X_{-3}	$(-12.6, 0.797)$
X_{-2}	$(-8.4, 0.353)$
X_{-1}	$(-4.2, 0.088)$
X_0	$(0, 0)$
X_1	$(4.2, 0.088)$
X_2	$(8.4, 0.353)$
X_3	$(12.6, 0.797)$
X_4	$(16.8, 1.421)$
X_5	$(21, 2.230)$
X_6	$(25.2, 3.227)$

[0019] Table 1

[0020] The advent in Dense Wavelength Division Multiplexing (DWDM) optical communication networks, however, requires that the multiple wavelengths in an optical fiber to be analyzed by spectral analysis devices that are much smaller in size than that of the current grating spectrometer. The challenge is to circumvent the current limitation in grating spectrometer design and fabrication methods. As discussed above, the current design basically cannot achieve the Resolution-Size factor (RS) much smaller than about 10. While several current technologies are capable of using planar waveguide technologies to make grating based spectrometers on a single silica or semiconductor substrate, they are still not able to achieve RS much smaller than 10 due to the basic limitations of the grating spectrometer design. Achieving a smaller RS factor is important for combining or integrating high-resolution grating spectrometers with various photonic devices (such as lasers, modulators, or detectors in a compact module or silica/silicon/semiconductor wafer).

[0021] These wavelength-division-multiplexed (WDM) integrated photonic devices or modules would be of great importance for applications to DWDM networks. The costs of these integrated WDM devices are typically proportional to

their sizes. The wavelength dispersion elements, such as the grating spectrometer or other form of wavelength filters, are typically about 100 times larger in size than any other photonic devices in the module or wafer. In order to reduce their costs substantially, it is desirable to reduce the size of these wavelength dispersion elements to as small a size as possible.

[0022] Thus, it is desirable to have grating based spectrometers that have an RS factor of less than 10. it is also desirable to reduce the size, and hence the cost, of integrated WDM devices that are used in DWDM networks. The present invention discloses such a device and a method for making the same.

SUMMARY OF INVENTION

[0023] It is an aim of the invention to provide a compact curved grating and associated compact curved grating spectrometer that is capable of achieving very small RS factors thereby enabling high resolution at small size.

[0024] It is another aim of the invention to provide a compact curved grating spectrometer module that can be used as an isolated optical spectrometer using discrete optical components.

[0025] It is another aim of the invention to provide a compact

curved grating spectrometer module that can be used as a wavelength dispersion element in a photonic integrated circuit.

[0026] In order to attain the above-mentioned aims, a compact curved grating and associated compact curved grating spectrometer is provided. The compact curved grating spectrometer includes an entrance slit, a detector and a curved grating. The locations of the entrance slit and the detector can be adjusted to control the performance of the spectrometer. The distance between the grooves of the gratings depend on the location of the entrance slit, the detector, the center of the operation wavelength, the diffraction order and the refractive index of the medium.

BRIEF DESCRIPTION OF DRAWINGS

[0027] The preferred embodiments of the invention will hereinafter be described in conjunction with the appended drawings provided to illustrate and not to limit the invention, wherein like designations denote like elements, and in which: FIG. 1A and FIG. 1B show different views of a curved grating having the Rowland design, and FIG. 1C shows ray-tracing for a Rowland grating design indicating focusing distortion or aberration at the exit slit for the cases where the input divergence angles are 4DEG (left),

8DEG (middle), and 16DEG (right); FIG. 2 shows the general specification of a curved-grating spectrometer; FIG. 3 illustrates the Rowland configuration specification for a Rowland curved grating; FIG. 4 shows a specific case of Rowland design curved grating; FIG. 5A describes the Comparison of angular resolution for Rowland grating (left) with input divergence angle 16DEG and the HR-CCG design with large-angle aberration correction at input divergence angle 50DEG (right); FIG. 5B describes High Resolution Compact Curved Grating specifications in accordance with a preferred embodiment of the present invention; FIG. 6 illustrates an example of High Resolution Compact Curved Grating with constant angle; FIG. 7 shows a High Resolution Compact Curved Grating with Constant Arc, the detector and the entrance slit being present on a tangent circle, in accordance with an embodiment of the present invention; and FIG. 8 shows a comparison between the Rowland design and the High Resolution Compact Curved Grating with constant groove (arc) length in accordance with the present invention.

DETAILED DESCRIPTION

[0028] The present invention discloses a system comprising a compact curved grating (CCG), its associated compact

curved grating spectrometer (CCGS) module and a method for making the same. The system is capable of achieving very small (resolution vs. size) RS factor. The uses of CCGS module include an isolated optical spectrometer using discrete optical components such as slits, grating, spectrometer casing, detector, detector array, and motor drive. More generally, the CCGS module could also be used as a wavelength dispersion element in a photonic integrated circuit. The photonic integrated circuit can be based on either of glass (silica) waveguide, semiconductor waveguide (including but not limited to, polymer waveguide, or any other type of optical waveguiding devices. Semiconductor waveguides include silicon or compound semiconductor waveguides such as III-V (GaAs, InP etc). The wavelength dispersion element based on the CCGS module in the photonic integrated circuit can be integrated with optical detector, laser, amplifier, modulator, splitter, multimode interference devices, other wavelength filters, array-waveguide-based devices, and other photonic devices, materials, or components to achieve a multi-component photonic integrated circuit with useful functionalities. The CCG explained below is a High Resolution Compact Curved Grating (HR-CCG) that tries to alle-

viate the disadvantages associated with prior art mentioned earlier, by providing a high resolution in a small (compact) module.

[0029] We have improved on the current Rowland design, enabling curved-grating spectrometer with 10–100x smaller linear size (or 100–10,000x smaller area) using our HR-CCG with large-angle aberration-corrected design. The typical Rowland design can only reach a useful diffraction angle Θ_{diff} of ~4DEG, beyond which serious aberration in the refocusing beam will occur to limit wavelength resolution. In Figure 5A we show the angular resolution of the typical Rowland design at 16DEG diffraction angle compared with our HR-CCG design at 50DEG. We see that our "large-angle aberration-corrected grating" design has much better angular resolution: different direction rays are well converged to a point on the focal circle. This translates to much smaller RS factor or size. We have used discrete time solution of vectorial Maxwell's equations to simulate the HR-CCG design, which verified the high resolution nature of our grating as predicted by the ray-tracing method.

[0030] FIG. 5B shows an embodiment of a HR-CCG and an associated spectrometer utilizing the HR-CCG. The HR-CCG

spectrometer should have the geometric configuration as described below.

[0031] First, the location of entrance slit 502 can be adjusted in order to have the best performance for a particular design goal. Thus, the location of a entrance slit 502 specified by angle θ_1 with respect to the normal of grating center 504 and the distance S_1 from grating center 504 is not necessarily on a circle as in the case for Rowland design mentioned in the prior art.

[0032] Second, the location of detector can be adjusted in order to have the best performance for a particular design goal. Thus, the location of detector 506, specified by the angle θ_2 with respect to the normal of grating center 504 and the distance S_2 from the grating center is not necessarily on the same circle where entrance slit 502 is located, nor on any other circle.

[0033] Third, The relation between θ_1 , θ_2 , and the initial groove spacing d is given by the grating formula,

[0034]
$$d(\sin \theta_2 - \sin \theta_1) = m \lambda_c / n \quad (1)$$

[0035] where m is the diffraction order, n is the refractive index of the medium, and λ_c is the center of the operation wavelength.

[0036] Fourth, initial groove positions are $X_0 = (0, 0)$, $X_1 = (d, R -$

$(R^2 - d^2)^{1/2}$) and $X_{-1} = (-d, R - (R^2 - d^2)^{1/2})$ With these position vectors, three initial grooves are located on a circle of radius R and have the initial groove spacing of d at the grating center.

[0037] Fifth, location of other grooves X_i 's are obtained by two conditions. The first of these conditions being that the path-difference between adjacent grooves should be an integral multiple of the wavelength in the medium. The first condition can be expressed mathematically by:

$$[d_1(\theta_1, S_1, X_i) + d_2(\theta_2, S_2, X_i)] - [d_1(\theta_1, S_1, X_{i-1}) + d_2(\theta_2, S_2, X_{i-1})] = m\lambda/n,$$

[0039] (2)

[0040] where $d_1(\theta_1, S_1, X_i)$ is the distance from a i-th groove located at X_i to entrance slit 502 specified by θ_1 and S_1 , $d_2(\theta_2, S_2, X_i)$ is the distance from a i-th groove located X_i to detector 506 specified by θ_2 and S_2 , m is the diffraction order, and n is the refractive index of the medium. This mathematical expression is numerically exact for the optical path difference requirement in the diffraction grating and is actively adjusted for every groove on HR-CCG.

[0041] The second of these conditions being specific for a particular design goal of a curved-grating spectrometer. The second condition in general can be mathematically ex-

pressed as

[0042] $f(\theta_1, S_1, \theta_2, S_2, X_i, X_{i-1}, \lambda_c, n, m) = \text{const} \quad (3)$

[0043] Specific examples of the second condition are described later in the application. The unknown real variables in both equations (2) and (3) are x - and y - coordinates of the location vector X_i of the i -th groove. For given input-slit location (θ_1, S_1) , detector 506 location (θ_2, S_2) , and the previous, i.e. $(i-1)$ -th, groove position X_{i-1} , X_i is completely specified by equations 3 and 4 for a given center wavelength λ_c , refractive index n , and the diffraction order m .

[0044] The last of the HR-CCG specification ensures that every ray from each groove focuses to a single point. This ensures HR-CCG having a large acceptance angle, and therefore a small spot size.

[0045] An exemplary embodiment of HR-CCG specified above is shown in FIG. 6. The radius of curvature at the grating center is $R = 50 \mu\text{m}$. Entrance slit 502 is located at an angle $\theta_1 = 55^\circ$ from the grating normal and distance $S_1 = 28.68 \mu\text{m}$ from the grating center. Detector 506 is located at an angle $\theta_2 = 27.2^\circ$ the grating normal and distance $S_2 = 37.65 \mu\text{m}$ from the grating center. The groove spacing at the grating center is chosen to be $d = 3.6 \mu\text{m}$ so that diffraction order $m = 10$ is directed toward detector 506

located at θ_2 . As shown in FIG. 6, entrance slit 502 and detector 506 is not located on a circle tangent to the grating center. Three initial grooves are located at $X_0 = (0, 0)$, $X_1 = (3.6, 0.13)$, and $X_{-1} = (-3.6, 0.13)$ which form a circle of radius $R = 50 \mu\text{m}$. Other groove locations X_i 's are obtained with the condition of each groove having a constant angular spacing from entrance slit 502 and optical path-difference condition (Eq. 2). In a mathematical form, this condition can be expressed as ,

$$\cos(\Delta\theta_i) = \frac{(X_i - X_{in}) \cdot (X_{i-1} - X_{in})}{|X_i - X_{in}| |X_{i-1} - X_{in}|} = \text{const}$$

[0046] (4)

[0047] where $X_{in} = (-S_1 \bullet \sin\theta_1, S_1 \bullet \cos\theta_1)$ is the position vector of entrance slit 502, $X_{det} = (-S_2 \bullet \sin\theta_2, S_2 \bullet \cos\theta_2)$ is the position vector of detector 506, and $\Delta\theta_i$ is difference in angular position between successive i^{th} and $(i-1)^{\text{th}}$ grooves. In Eq. 4, operator " \bullet " means the inner product in vector analysis and defined as $A \bullet B \equiv |A| |B| \cos\theta$. Because $\Delta\theta_i$ is constant for all grooves, it is same as the angular-position difference between the center groove at X_0 and the first groove at X_1 , i.e.

[0048]

$$\Delta\theta_1 = \arccos \frac{(X_1 - X_{in}) \cdot (X_0 - X_{in})}{\|X_1 - X_{in}\| \|X_0 - X_{in}\|}$$

[0049] . In this particular case, the position of entrance slit 502, exit slit 506 and the angular spacing between the grooves are $X_{in} = (-23.49, 16.45)$, $X_{det} = (-17.26, 33.46)$, and $\Delta\theta_1 = 4.13^\circ$. In this example, wave-front of the diverging input beam propagating toward the curved grating is sliced into a set of narrow beams with angular extension $\Delta\theta$ by the curved-grating. Each beam with angular extension $\Delta\theta$ undergoes reflective diffraction by each groove. At a particular wavelength, diffraction at a particular groove is equivalent to redirecting to a particular narrow beam into a detector 506 location with θ_2 . The position vectors X_i 's calculated from Eq. (2) and Eq. (4) are listed in the Table 2. As shown in FIG. 6, the positions of grooves X_i are not on a circle tangent to grating.

[0050]

X_{-10}	(-23.17, 15.28)
X_{-9}	(-22.24, 12.89)
X_{-8}	(-20.97, 10.60)
X_{-7}	(-19.36, 8.43)
X_{-6}	(-17.42, 6.44)
X_{-5}	(-15.17, 4.65)
X_{-4}	(-12.62, 3.10)
X_{-3}	(-9.80, 1.83)
X_{-2}	(-6.74, 0.87)
X_{-1}	(-3.60, 0.14)
X_0	(0.00, 0.00)
X_1	(3.60, 0.14)
X_2	(7.30, 0.70)
X_3	(11.06, 1.70)
X_4	(14.83, 3.13)
X_5	(18.57, 5.02)
X_6	(22.22, 7.36)
X_7	(25.73, 10.16)
X_8	(29.06, 13.39)
X_9	(32.16, 17.06)
X_{10}	(34.98, 21.15)

[0051] The above example has been used for illustration purposes only and should not be construed in any way as limiting the scope of the invention.

[0052] In an alternate embodiment, the High-Resolution Compact Curved Grating has Constant Arc and the Detector is located on a tangent Circle. This embodiment is described below in detail.

[0053] In this exemplary embodiment, both entrance slit 502 and detector 506 are located on a circle tangent to the grating center as in the case of Rowland design mentioned earlier. However, grooves in this curved-grating are located such that the arc-length of each groove is the same. As a result, grooves are not located on a circle nor are spaced with equal distance.

[0054] There are two commonly used shapes of grooves in the grating used in the free-space spectrometer. They are straight line and sinusoidal shape. These two shapes are widely used because of the ease of manufacturing process. For a curved-grating, ideal shape of the reflecting surface is not a straight line, but a curved shape that can image entrance slit 502 at detector 506 location. Ideal aberration-free curved mirror is an ellipse with its focal point located at source and image. Therefore, the ideal shape of the groove in a curved-grating is a section of ellipse with its focal points at the slit and the detector. In this embodiment, elliptic shape is used for each groove and the length of this elliptic shape in each groove is kept constant. Center positions of the grooves X_i 's in this example are determined so that the length of each elliptic groove is the same.

[0055] The geometric specification of the HR-CCG with constant arc-length and detector 506 at a tangent circle is as described below.

[0056] First, entrance slit 502 is located on a circle tangent to the grating at its center (so-called tangent circle). Therefore, the angle θ_1 and the distance S_1 of entrance slit 502 with respect to the grating center is related by $S_1 = R \cos \theta_1$, where R is the radius of curvature of the grating center.

[0057] Second, like entrance slit 502, detector 506 is located on a circle tangent to the grating at its center. Therefore, the angle θ_2 and the distance S_2 of detector 506 with respect to the grating center is related by $S_2 = R \cos \theta_2$, where R is the radius of curvature of the grating center.

[0058] Third, the relation between θ_1 , θ_2 , and the initial groove spacing d is given by the grating formula, $d(\sin \theta_2 - \sin \theta_1) = m \lambda_c / n$ where m is the diffraction order, n is the refractive index of the medium, and λ_c is the center of the operation wavelength.

[0059] Fourth, initial groove positions are $X_0 = (0, 0)$, $X_1 = (d, R - (R^2 - d^2)^{1/2})$ and $X_{-1} = (-d, R - (R^2 - d^2)^{1/2})$ With these position vectors, three initial grooves are located on a circle of radius R and have the initial groove spacing of d at the grating center.

[0060] Fifth, the location of other grooves X_i 's are obtained by the following two conditions. The first condition being the path-difference between adjacent grooves should be an integral multiple of the wavelength in the medium, which is mathematically expressed as

$$[d_1(\theta_1, S_1, X_i) + d_2(\theta_2, S_2, X_i)] - [d_1(\theta_1, S_1, X_{i-1}) + d_2(\theta_2, S_2, X_{i-1})] = m\lambda/n,$$

[0062] (2)

[0063] The arc-lengths of all the grooves are the same throughout the HR-CCG. This condition can be mathematically expressed as

$$\Delta S_i = |X_i - X_{in}| \arccos \frac{\left(\frac{X_{i-1} + X_i}{2} - X_{in} \right) \cdot \left(\frac{X_i + X_{i+1}}{2} - X_{in} \right)}{\left| \frac{X_{i-1} + X_i}{2} - X_{in} \right| \left| \frac{X_i + X_{i+1}}{2} - X_{in} \right|}$$

[0065] (5)

[0066] where ΔS_i is the arc-length of i^{th} groove. This equation requires the knowledge of X_{i+1} , which is still unknown. However, with the constraint the fact that each X_i is located at the center of the groove, the above expression can be reduced to the following expression without X_{i+1} .

[0067]

$$\Delta S_i = 2|X_i - X_{in}| \arccos \frac{\left(\frac{X_{i-1} + X_i}{2} - X_{in} \right) \cdot (X_i - X_{in})}{\left| \frac{X_{i-1} + X_i}{2} - X_{in} \right| |X_i - X_{in}|} = \text{const}$$

[0068] (6)

[0069] FIG. 7 shows a specific example of the HR-CCG with constant arc-length of the grooves and detector 506 at a tangent circle. The radius of curvature at the grating center is $R = 100 \mu\text{m}$. Entrance slit 502 is located at an angle $\theta_1 = 45^\circ$ from grating normal and a distance $S_1 = 70.71 \mu\text{m}$ from the grating center. Detector 506 is located at an angle $\theta_2 = 37.37^\circ$ and distance $S_2 = 79.47 \mu\text{m}$ from the grating center. Both entrance slit 502 and exit slit 506 are located on a tangent circle of radius $50 \mu\text{m}$. The groove spacing at the grating center is chosen to be $d = 4.2 \mu\text{m}$ so that diffraction order $m = 12$ is directed toward detector 506 located at an angle θ_2 from the grating normal. Three initial grooves are located at $X_0 = (0, 0)$, $X_1 = (4.2, 0.008)$, and $X_{-1} = (-4.2, 0.008)$ which form a circle of radius $R = 100 \mu\text{m}$. Other groove locations X_i 's are obtained with the condition of arc-length of each groove ΔS_i is the same, i.e. $\Delta S_i = \Delta S_1$. Equation (2) and (6) are simultaneously solved for a X_i with $X_{in} = (-50, 50)$, $X_{det} =$

$(-48.24, 63.15)$, and $\Delta S_1 = 4.201 \text{ } \mu\text{m}$ for a given X_{i-1} .

Groove locations, X_i 's calculated in this method are listed in Table 3. As shown in FIG. 7, grooves in this grating are not located on a tangent circle.

[0070]

X_{-15}	(-55.43, 23.48)
X_{-14}	(-52.90, 20.32)
X_{-13}	(-50.07, 17.38)
X_{-12}	(-46.98, 14.68)
X_{-11}	(-43.67, 12.21)
X_{-10}	(-40.17, 9.98)
X_{-9}	(-36.52, 7.99)
X_{-8}	(-32.74, 6.24)
X_{-7}	(-28.84, 4.72)
X_{-6}	(-24.86, 3.43)
X_{-5}	(-20.81, 2.35)
X_{-4}	(-16.71, 1.48)
X_{-3}	(-12.57, 0.82)
X_{-2}	(-8.39, 0.36)
X_{-1}	(-4.20, 0.09)
X_0	(0.00, 0.00)
X_1	(4.20, 0.09)
X_2	(8.39, 0.35)
X_3	(12.57, 0.77)
X_4	(16.73, 1.34)
X_5	(20.86, 2.07)
X_6	(24.97, 2.94)
X_7	(29.04, 3.96)
X_8	(33.07, 5.10)
X_9	(37.06, 6.37)
X_{10}	(41.01, 7.76)
X_{11}	(44.91, 9.28)
X_{12}	(48.77, 10.90)
X_{13}	(52.57, 12.63)
X_{14}	(56.33, 14.47)

[0071] The above example has been used for illustration purposes only and should not be construed in any way limit-

ing the scope of the above-described embodiment or invention as a whole.

[0072] The performance of the HR-CCG with the constant arc-length and detector on a tangent circle is compared with a Rowland design with the same parameters such as θ_1 , S_1 , θ_2 , S_2 , R , m , d , and λ_c . It is a direct comparison of a Rowland curved-grating spectrometer described in FIG. 3 and a HRC-curved grating spectrometer described in FIG. 7. All the configuration parameters are the same for these two spectrometers except the grating itself. Particularly, the imaging properties, that is, how well entrance slit 502 is sharply imaged at the detector location without aberration are compared. Imaging properties ultimately determine the resolution of a spectrometer. Finite Difference Time Domain (FDTD) method is used as a calculation method. FDTD is a Maxwell-equation solver, which evaluates electromagnetic wave within a spatial region for a certain period of time. By choosing a fine spatial grid size and temporal calculation step, the equation for an electromagnetic wave and its propagation can be solved with arbitrarily fine resolution. Imaging properties in these two curved-grating spectrometers is calculated by propagating a monochromatic light into entrance slit 502 of each

spectrometer. FDTD is run until the interference of beams from the entire grating groove is completed and forms an image of entrance slit 502 at the detector location. The resulting snapshot of electric-field near the detector is taken for these two cases as shown in FIG. 8. Entrance slit 502 width of $1\mu\text{m}$ is used for both simulations and the wavelength of $\lambda=1530, 1550, 1570\text{nm}$ is used. In FIG. 8A shows the snapshot of electric field at the detector location for the Rowland design described in FIG. 3. As expected, the image of the entrance slit is blurred due to imperfect grating. For $1\mu\text{m}$ entrance slit, the full diffraction angle is about $\theta_{\text{div}} = 50^\circ$ and therefore, Rowland design breaks down. Figure 8B shows the snapshot of electric field for the HR-CCG with constant arc-length grooves and detector on a tangent circle. In this case, a sharp aberration free image of entrance slit is formed at the detector location. The RS factor ($RS = (\Delta\lambda/\lambda) \times (L/\lambda)$) in this case is 0.6.

[0073] In an alternate embodiment, High-Resolution Compact Curved Grating has a constant arc with the detector being present in-line. With reference to FIG. 7, this embodiment can be realized if the input and exit slits are located along a line such that $S1 \approx S2$. In another alternate embodiment,

High-Resolution Compact Curved Grating has a constant arc with detector 506 present at an arbitrary location.

[0074] In another alternate embodiment, High-Resolution Compact Curved Grating has a constant angle and detector 506 is present on the circle of radius R , as depicted in FIG. 6. In this embodiment, each groove surface has an angular extension $(\Delta\theta)_i$ from entrance slit 502. In this example, the angular extensions $(\Delta\theta)_i$ are kept constant for all grooves. In addition, both entrance slit 502 and detectors 506 are located on a circle of radius $R/2$, where R is the radius of a circle formed by three initial groove locations X_0 , X_1 , and X_2 .

[0075] While the preferred embodiments of the invention have been illustrated and described, it will be clear that the invention is not limited to these embodiments only. Numerous modifications, changes, variations, substitutions and equivalents will be apparent to those skilled in the art without departing from the spirit and scope of the invention as described in the claims.